Tuning of the Equilibrated Residual Method for Applications in Direct, Inverse and General Piezoelectricity

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ABSTRACT

This paper presents application of the Equilibrated Residual Method (ERM) of a posteriori error estimation to the problems of direct, inverse and general piezoelectricity. In this three cases, either electric potential is induced by strains or strains appear due to the applied electric potential or both phenomena occur simultaneously. The first two cases correspond to the so-called sensing and actuating modes of piezoelectric transducers’ operation, while the third one to the mixed mode. The mentioned ERM is applied to the error assessment of the numerical finite element solution. The error values usually serve as indication for adaptive hierarchical modeling (within thin structures of thickness \(t\)) and adaptive mesh changes of \(hp\)-type within thin and solid structures where \(h\) and \(p\) stand for the generalized mesh (or element) dimension parameter and the element approximation order, respectively.

The described above application of the Equilibrated Residual Method for adaptivity has been demonstrated for various elliptic problems. In the context of piezoelectricity, such elliptic cases correspond to pure linear elasticity within elastostatics and pure linear dielectricity within electrostatics. Note that pure elasticity and dielectricity can be coupled through the electro-mechanical coupling by means of piezoelectric constitutive constants in order to form the coupled problem of piezoelectricity. In the case of elliptic problems the \(t\)-convergence and \(hp\)-convergence theories exist and the upper bound property of the ERM error estimation can be demonstrated. Thanks to these theories and property, the relation between the hierarchical model \(q\) and discretization parameters \(h\) and \(p\) and the estimated modeling and approximation errors can be established and the error-controlled model and mesh adaptation can be performed. The quality of error estimation with ERM depends on the proposed local residual problems and is different for the mechanical and electric problems. In the local problems, usually linear equilibration at element vertex nodes is applied as a standard. It turns out, however, that such a standard may not be sufficient for high quality estimation, where the modeling and approximation error effectivity indices (ratios of the estimated and true error values) close to one are required. In such cases, tuning of ERM local problems is necessary which may include additional higher-order equilibration at higher-order nodes and/or complete constraining at vertex nodes instead of standard rigid motion elimination.

In the case of the coupled problems of piezoelectricity, the ineffective error estimation partly results from the inherent definition of the standard ERM which proposes linear equilibration and rigid motion elimination for the mechanical field of displacement and the analogous equilibration and elimination for the electric potential field. Additionally, in the case of the coupled problems, the upper bound property of the estimation cannot be proved and the convergence theories typical for elliptic problems do not apply. In such circumstances, the effectivity of ERM is much worse than for the pure elliptic problems. Moreover, this effectivity very much depends on the piezoelectricity problem under consideration, i.e. it is different for direct, inverse and general piezoelectricity. It is also different for the mechanical, electric and coupling energies. The methods of obtainment of acceptable quality of the estimation for the piezoelectric problems are presented in the paper.

Keywords: Adaptivity, Error estimation, Equilibrated Residual Method, Finite Element Method, Hierarchical modeling, Piezoelectricity.